

“continuous action”) is a good example. Its two ingredients are in fact PPA and CPM, and the recommended dosing interval is 12 hours rather than 6 hours. An interesting series of exercises can be built by assuming different, nonuniform sustained-release patterns.

Another interesting set of exercises can be built around lithium, which is used for treatment of manic-depressive illness. Its half-life in the GI tract is approximately 1/2 hour, and its half-life in the blood is approximately 24 hours. Lithium (carbonate, chloride, etc.) is usually given in immediate release form, with a 12 hour dosing interval. As opposed to most drugs, lithium has a low ratio of toxic to therapeutic levels. Because of the long half-life, there will be an accumulation over several days as the blood levels build to steady state. Some care must be taken that the blood levels do not ultimately rise above the threshold of toxicity.

For a patient with active manic-depressive disorder, the physician might prescribe an initial loading dose to bring blood levels more rapidly to steady state. How large a loading dose can be given without causing the blood levels ever to rise above the steady state (peak) levels? Elderly patients often have impaired renal function, leading to longer medication half-life. If a patient has a half-life of 36 hours for lithium in the blood but all else remains the same, how much larger do the steady state levels become? What happens if the half-life is 48 hours?

**Generalizations.** Pharmacokinetics goes far beyond the simple two-compartment models we have dealt with here. Depending on the drug, different organs of the body may store substantial fractions of the total volume. In fact, in many investigations, the GI tract is bypassed

by injecting the drug directly into the blood, and still there may be as many as four compartments involved. With high concentrations and/or clearance by metabolism, the differential equations may become nonlinear.

There is considerable literature on these topics; I did a Medline search from 1986 to the present for abstracts containing both of the words “lithium” and “pharmacokinetic(s)” and found 66 different journal articles. Consider what I have presented here to be just a scratch on the surface of a very deep subject!

#### References:

- Godfrey, Keith, **Compartmental Models and Their Application**. London, New York: Academic Press, 1983.
- Welling, Peter G., **Pharmacokinetics: Processes and Mathematics**. Washington: American Chemical Society, 1986. □

## Teaching ODEs With Computer Experiments

Harvey Mudd College Workshop  
June 1992

**Robert Borrelli**

The 5-day workshop at Harvey Mudd College had 36 participants from June 16 thru the 20th, 1992. The workshop was supported by a grant from the Undergraduate Faculty Enhancement Program through the NSF Division of Undergraduate Education. The main speakers were Robert Borrelli and Courtney Coleman; a number of short talks by the participants gave added perspective to the workshop.

Two Macintosh labs were available to all participants on a 24-hour basis, as well as a PC lab and a VAX workstation lab running a broad variety of (mostly) commercial ODE solvers (or packages that contain ODE solvers). The main goal of the workshop was to give participants a glimpse of how computer experiments can be designed to support instruction in ODE courses. Exposure to a wide variety of textbooks and hardware/software platforms along the way was an added bonus. A preprint of Borrelli, Coleman and Boyce's **Differential Equations Laboratory Workbook** was sent in advance and free of charge to all participants by John Wiley & Sons.

**The Participants.** The largest contingent (18) came from California (of course) with 3 from Arizona, 2 from Missouri, and one each from Washington, Maine, Colorado, Wisconsin, Maryland, North Carolina, New Jersey, Texas, Florida, Kansas, Virginia and Utah. NSF funds were not sufficient to support all these participants; some opted to attend without NSF support (as did one participant from Mexico). Three participants were authors of ODE textbooks and monographs, and at least three have "published" their own ODE solvers. Eight participants were from two-year colleges, 13 from four-year colleges, and 15 from universities. One participant was from an electrical engineering department, the others all from mathematics departments.

**The Labs.** *The Mac Labs* contain fifteen Macintosh IIsi computers and eight Macintosh IIci computers connected on an ethertalk network and all using a single Macintosh Quadra 900 as a file server. The Macintosh IIci machines are equipped with large NEC 5G color moni-

tors. A LaserWriter IIg was shared by all machines on the network. The labs were running **MacMath 9.x**, **Differential Systems 3.0**, **MatLab 3.5**, **Maple V**, **Mathematica 2.0**, and **HiQ**. Only the IIci machines were equipped with the memory and CPU required to run **Mathematica 2.0**; the other software could be run on any machine. The largest difficulties came from the fact that while the machines were reserved for workshop use during the day, they were fully accessible to other students and faculty throughout the week. This arrangement resulted in occasional downtime.

*The PC Lab* supports nine IBM personal system/2 model 30 computers, each connected to an IBM personal system/2 model 70 using Novell network software. Installed on these are **Phaser 1.1**, **MatLab 3.5**, **Maple V** and **MDEP**. One problem noted was that Phaser is not written to be used on a network and needed to be put on the hard drives of each model 30 computer.

*The VAX Workstation Lab* is a cluster of five 3100s, 2 model 30s and 2 model 76s, and one SPX with a color monitor, all under the VMS operating system running **ODETOOLKIT**, a menu-driven adaptive ODE solver package under local development. **ODETOOLKIT** can be accessed from any machine on campus. The network design and the current state of development of the solver package combined to make **ODETOOLKIT** rather slow when used by more than three users at a time.

**Outcome.** A questionnaire distributed at the end of the workshop revealed that it was overwhelmingly successful in achieving its goals, in spite of the intermittent air conditioning and uncooperative hardware. An often expressed sentiment was appreciation of the oppor-

tunity to share experiences with colleagues from diverse teaching environments, while having access to a variety of software and textual materials. One participant later said that:

*I've made arrangements to introduce class demonstrations using a computer in my numerical analysis class (taught on television) this Fall. Our chair received the software from Arizona: we are discussing how best to use the **Are You Ready For...** programs. Next Spring I'll definitely teach an ODE class using **MDEP** for the computer experiments. My experience at the Workshop has reinforced our intentions to experiment with the use of graphing calculators in Calculus and Phaser in a course on Dynamical Systems this year.* □

## Teaching ODEs With Computer Experiments

Cornell Workshop, June 1992  
Anne Noonburg

The workshop was held at Cornell University from May 28 to June 3, 1992, and was run by Beverly West, John Hubbard, and Anne Noonburg. It brought together a diverse group of 35 participants, teachers from community colleges, small liberal arts colleges, as well as from large research universities. Their experience varied widely, from some who had never used computers in their teaching to a few who had already writ-

ten sophisticated differential equations software. In spite of this wide variation in background, these people worked extremely well together to produce some exciting new projects for teaching differential equations.

The conference schedule was planned to give the participants the opportunity to: find out what is already available for teaching differential equations with computer experiments; learn how to use the Macintosh platform, and the Cornell software, in particular (*Ed: MacMath 9.0*<sup>1</sup>); discuss with the conference organizers and with each other how computers may change the teaching of differential equations; produce -- in small teams -- major projects to be used as computer experiments for their own classes, as well as to serve as ideas for other differential equations instructors not able to attend these conferences.

Several of the participants had backgrounds in scientific fields outside of mathematics, and this led to interesting cross-disciplinary discussions. John Hubbard gave three early morning lectures, and several participants commented that this was one of the best aspects of the conference. In their evaluations collected at the end of the conference, many people expressed the feeling that it was good to meet and exchange ideas with colleagues from across the country. As one participant put it:

*For the first time, I was part of a group of teachers of differential equations who overwhelmingly believe in qualitative analysis and in serious efforts to have students understand what differential equations are about.*

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