(Editor's Note: There has been a recent resurgence of interest in the Painlevé transcendents and their solutions because of their utility in modeling nonlinear phenomena. As you can see from the figures, the solution curves of the first transcendent are not only interesting, but look very nice!) ❑

Computing Changes Core Mathematics

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The New Core: The curriculum at USMA provides four semesters of core mathematics given to all students during their first two years. Using the recent advances in bringing computing and experimenting into the classroom, USMA has devised a new core mathematics curriculum. This new curriculum has been in place since Fall 1990 and was designed using a systems engineering, top-down approach which tries to fit material from several desired topics courses into the following four course sequence: **Discrete Dynamical Systems** (difference equations), **Calculus I** (differential and integral), **Calculus II** (multivariable), and **Probability and Statistics**.

The concepts and topics from differential equations and matrix algebra are spread through the first three courses. Mathematical modeling is encountered during all four semesters. This article presents some of the details of how this was done.

Computation and the Core: USMA is fortunate to have a rich computing environment for mathematics education. All students are required to purchase a personal computer, a computer algebra system (**Derive**), a spreadsheet program (**Quattro-Pro**), a statistical package (**Minitab**), and an advanced symbolic/graphing calculator. These tools are fully integrated into the curriculum and are used by students to learn mathematical process by discovery.

The underlying concepts of discrete behavior, functions, and models are presented in the context of difference equations during the first course. In this course, the computer is used as a tool to perform iteration, to conduct numerical experiments, to analyze behavior, and to plot solutions. With this conceptual and computational foundation, calculus becomes a way to model and analyze differential equations, the continuous analog of difference equations.

The new curriculum helps the students form a different relationship to mathematics. Some of the ideas that are nurtured by the new curriculum include: *i)* Mathematics is a medium of communication whose ideas are structured in mathematical format and can be expressed with numbers, graphs, or symbols . *ii)* Curiosity and a disposition to experiment are essential. *iii)* Mathematics is useful and interesting.

Specific objectives within the curriculum strengthen student development in the following areas: Understand the basics of the processes of discrete and continuous mathematics, the differences between linear and nonlinear mathematics, and the contrasts between stochastic and deterministic mathematics. Acquire the ability to compute and then communicate mathematically. Use analytic, graphical, and numerical approaches to problem solving. Develop the ability to formulate questions, to seek appropriate references, and to understand mathematical writing.

Linear & Nonlinear, Discrete & Continuous: Most of the models presented in undergraduate courses to date have been designed to be linear, deterministic, and smooth. The world is often nonlinear, however, and phenomena are commonly random, ill-behaved, and discrete. The increasing power of the computer has made it much less necessary to assume continuous or deterministic behavior in order to perform analysis and find solutions.

CAS: Computer algebra systems (CAS) can do many of the computations traditionally required of our undergraduates. Among other things, CAS packages can perform iteration, find derivatives and integrals, graph in two and three dimensions, manipulate matrices, determine Taylor polynomials, and solve systems of functional equations, difference equations, and differential equations. However, a CAS cannot set up the problem, conduct an experiment, interpret the solution, or check to see if the solution makes sense.

The New Core Courses: Each of the four core courses contains mathematics in symbolic, graphic, and numeric form. There is an emphasis on the understanding of the concept of a function in all the courses. Discovery and experimentation are encouraged and fostered, and communications is emphasized as part of the solution process.

Discrete Dynamical Systems emphasizes pattern recognition and problem solving. The modeling process is introduced and applies to example problems from the physical and social sciences. Given a problem, students learn to model changing behavior with difference equations and to determine symbolic, graphic, and numeric solutions. Spreadsheets and CAS are used to perform experiments and display data in order to find patterns in the model's behavior.

These discrete systems lend themselves to examination using iteration by a spreadsheet or CAS package. The students also learn basic matrix algebra for solving systems of difference equations and finding eigenvalues and eigenvectors. The behavior of nonlinear models is studied through numerical and graphical experimentation. This course provides practical modeling experience and is accessible from high school algebra. The need for continuous analogs to the discrete structures and processes found in this course provide an orderly, logical, and motivational transition to the study of both calculus and differential equations.

Calculus I and **Calculus II** are taught with full appreciation of the computational resources available to the students. The transition from discrete to continuous systems is motivated by the need to reduce to zero the size of the step (or change) in the difference equations which model continuous behavior. This illuminates the concepts of limit, derivative, and differential equations. Integration is similarly seen as a limit process involving discrete sums. The students then explore the applications of the calculus, using the computer to do the computations. By eliminating many of the routine computational techniques, we can cover more concepts, introduce more applications, and revisit applications that were previously modeled with discrete variables using continuous variables.

Probability and Statistics require the student to draw on the mathematics learned in the previous courses and require significant writing and computing skills. This course emphasizes communicating problem solutions through the use of a case study approach. Students make heavy use of the statistical package to draw inferences about their case problems with reliability and data analysis.

Why Change? The change in the curriculum was primarily driven by new opportunities for the students to perform calculations and computations. Computing and experimenting encourage more enthusiastic problem solving by students and improve their attitude about the usefulness of mathematics. The introduction of **Discrete Dynamical Systems** as the first course in collegiate mathematics (instead of calculus) is the most significant curriculum change.

Results: Does the new approach work? We think so, but ask us again in a year or two. ❑

The Flight of a Ski Jumper

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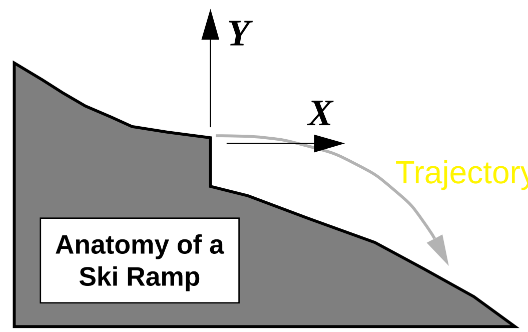
(Editor's Note: True outlines a laboratory experiment that will be a big hit with skiers. He also has a shorter version of the experiment; write him for details.)

Modeling the Forces: When a ski jumper leaves the ramp of a ski jump and becomes airborne, the dominant forces that determine the success of the jump are the force due to gravity and the air resistance, which can be resolved into a lift force and a drag force. The lift and drag forces can be altered by the jumper's posture over the skis and by the position of the skis relative to the ground. This combination of lift and drag can make the difference between a "good" jump and a short or even dangerous jump. In Figure 1 a coordinate system is placed

Figure 1



with the origin at the lip of the ramp where the jumper becomes airborne. To examine the forces, let $\vec{R}(t) = x(t)\hat{i} + y(t)\hat{j}$ denote the position vector of the jumper at time $t \geq 0$ (seconds). The velocity vector is then $\vec{V}(t) = x'(t)\hat{i} + y'(t)\hat{j}$ and points in the direction of motion at time t with speed $S = |\vec{V}(t)| = \sqrt{x'^2 + y'^2}$. We will assume that the jumper's path lies in the xy-plane and that local winds are negligible. Let's look at the forces.

a) The force due to **gravity**: $\vec{F} = -mg\hat{j}$, where $g = 32.2$ ft./sec^2, and m is the mass of the jumper including skis and boots.

b) The **lift** force, which we take to be perpendicular to the flight path of the jumper and with magnitude proportional to the jumper's speed, so that

$$\vec{L} = \lambda(-y'(t)\hat{i} + x'(t)\hat{j})$$

where λ is the lift coefficient.

c) The **drag** force, which opposes the direction of motion. We will assume that this force is proportional to the velocity ,

$$\vec{D} = -\delta(x'(t)\hat{i} + y'(t)\hat{j})$$

where δ is the drag coefficient.