statistical package to draw inferences about their case problems with reliability and data analysis.

Why Change? The change in the curriculum was primarily driven by new opportunities for the students to perform calculations and computations. Computing and experimenting encourage more enthusiastic problem solving by students and improve their attitude about the usefulness of mathematics. The introduction of **Discrete Dynamical Systems** as the first course in collegiate mathematics (instead of calculus) is the most significant curriculum change.

Results: Does the new approach work? We think so, but ask us again in a year or two. ❑

The Flight of a Ski Jumper

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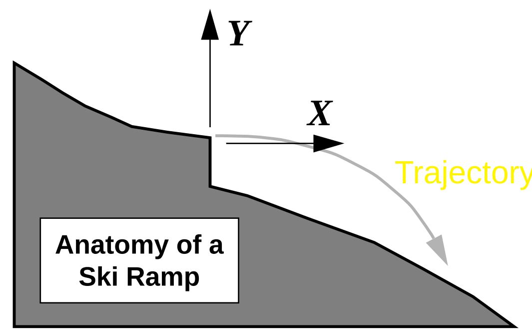
(Editor's Note: True outlines a laboratory experiment that will be a big hit with skiers. He also has a shorter version of the experiment; write him for details.)

Modeling the Forces: When a ski jumper leaves the ramp of a ski jump and becomes airborne, the dominant forces that determine the success of the jump are the force due to gravity and the air resistance, which can be resolved into a lift force and a drag force. The lift and drag forces can be altered by the jumper's posture over the skis and by the position of the skis relative to the ground. This combination of lift and drag can make the difference between a "good" jump and a short or even dangerous jump. In Figure 1 a coordinate system is placed

Figure 1



with the origin at the lip of the ramp where the jumper becomes airborne. To examine the forces, let $\vec{R}(t) = x(t)\hat{i} + y(t)\hat{j}$ denote the position vector of the jumper at time $t \geq 0$ (seconds). The velocity vector is then $\vec{V}(t) = x'(t)\hat{i} + y'(t)\hat{j}$ and points in the direction of motion at time t with speed $S = |\vec{V}(t)| = \sqrt{x'^2 + y'^2}$. We will assume that the jumper's path lies in the xy-plane and that local winds are negligible. Let's look at the forces.

a) The force due to **gravity**: $\vec{F} = -mg\hat{j}$, where $g = 32.2$ ft./sec^2, and m is the mass of the jumper including skis and boots.

b) The **lift** force, which we take to be perpendicular to the flight path of the jumper and with magnitude proportional to the jumper's speed, so that

$$\vec{L} = \lambda(-y'(t)\hat{i} + x'(t)\hat{j})$$

where λ is the lift coefficient.

c) The **drag** force, which opposes the direction of motion. We will assume that this force is proportional to the velocity ,

$$\vec{D} = -\delta(x'(t)\hat{i} + y'(t)\hat{j})$$

where δ is the drag coefficient.

The Model ODEs: Using Newton's second law of motion, $\vec{F} = m\vec{a}$, the components of the acceleration in the x and y directions are:

$$\begin{aligned} mx''(t) &= -\delta x'(t) - \lambda y'(t) \\ my''(t) &= \lambda x'(t) - \delta y'(t) - mg \end{aligned} \qquad (1)$$

If we divide by m and set $a = \delta/m$ and $b = \lambda/m$, we get

$$\begin{aligned} x''(t) &= -ax'(t) - by'(t) \\ y''(t) &= bx'(t) - ay'(t) - 32.2 \end{aligned} \qquad (2)$$

In this lab the students will draw some flight trajectories for the ski jumper on the computer for different values of a and b in the system (2). This will give them a collection of possible flight trajectories for different combinations of lift and drag, which can be controlled by the jumper and his or her equipment. Before we draw any flight trajectories, we will first rewrite the differential equations (2) in a more convenient form.

Exercise 1: The Initial Value Problem.

A. Notice that the two second order differential equations in (2) contain no terms involving $x(t)$ or $y(t)$. Integrate both sides of (2) with respect to t to obtain two first order differential equations.

$x'(t) =$

$y'(t) =$

B. You should now have two constants of integration in your equations which can be determined from the initial conditions. Since system (2) involves two second order equations, there are four ingredients in the initial conditions. Assume that the ski jumper leaves the lip of the ramp with an initial velocity of $\vec{V}(0) = 78\hat{i} + 0\hat{j}$. (thus the initial speed is 78 feet per second, or about 53 m.p.h.).

Write the four initial conditions for your system.

$x(0) =$ $\qquad$ $y(0) =$

$x'(0) =$ $\qquad$ $y'(0) =$

C. Using these initial conditions, determine the constants of integration for your equations in part A.

$x'(t) =$

$y'(t) =$ $\qquad$ (3)

Exercise 2: Flight Path.

The ski jumper's objective is to maximize the total distance travelled from the top of the takeoff ramp to setting down on the landing field. In this exercise you will use your equations (3) to draw some flight trajectories for different values of the drag and lift parameters a and b, as well as for different initial speeds. Although a and b actually change in value throughout the flight as the jumper adjusts the ski position, we will take them to be constant.

On this particular jumping hill, a marker is set at $x = 230$ feet and $y = -131$ feet. If the jumper comes near this point, he will be close to a jumping distance of about 265 feet, measured from the takeoff ramp down the hill.

At the computer, enter the system of two first order equations (3). For different values of a and b, you will draw flight trajectories in the xy-phase plane.

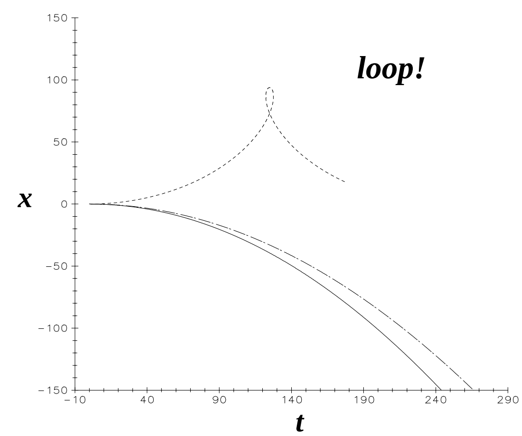
A. Set the phase plane window to $-20 \le x \le 300$ and $-200 \le y \le 20$. Try $a = 0.02$ and $b = 0.024$ in your equations and draw the flight trajectory over the time interval $0 \le t \le 3$. Save the trajectory and draw another with $a = 0.02$ and $b = 0.015$. Does this look like a better jump than the first? Again, save the trajectories and draw another with $a = 0.018$ and $b = 0.05$ and compare with previous jumps. Although the trajectories look similar, even slight changes in a and b can improve the jump distance by a few feet.

B. To determine the lengths of the jumps you have just drawn, you will need to look at the x, y coordinates when t is around 3 seconds. The ski jumper would like to get close to the point $x = 230$, $y = -131$ on the landing field. For comparison, look for a y-coordinate of $y = -131$. If $x < 230$ then the jumper fell short of the desired mark. Looking at these coordinates for each of the three choices for a and b in part A, determine the combination of a and b that will produce the best jump. The point in all this is that once an appropriate combination of a and b have been determined from this simulation, the jumper can then determine the body profile and angle of attack for the skis to produce the longest jump.

C. To determine the effect of the jumper's takeoff speed, change your equations so that the jumper's initial speed is increased from 78 ft./second to 85 ft./second. Compare the jumper's distance to the results above. Notice that an increase in takeoff speed means a better jump! Heavier jumpers usually get better takeoff speeds, but less lift, so somewhere there is a jumper with optimum weight.

D. ***"Hot Dogging!"*** The drag and lift parameters a and b are determined from the jumper's weight, profile, clothing, cross-sectional area, and ski position, so their values lie within a fairly small interval. To illustrate what can happen with "unrealistic" values, set the initial speed back to 78 ft./second and try $a = 0.0$ and $b = 0.9$. An increase in b means the jumper has more lift. Draw the jumper's flight trajectory with these values to see how ski jumping can lead to loops and ski flying. Set the window with $-20 \le x \le 200$ and $-50 \le y \le 100$ with $0 \le t \le 4$.

(Editor's note: when I got to this point in True's manuscript I had to try the experiment myself! Sure enough, lots of lift and no drag take the skier into loops (the dashed curve in the figure below). The other two flight paths correspond to the values of a, b, and initial speed given at the start of 2A and 2C, respectively. The time interval is set to $0 \le t \le 6$.)



E. Experiment with your equations to see what other flight trajectories can be obtained. Try values for $a > b$ and $a < b$. See if you can come up with a good lift to drag ratio, b/a.

F. By going back to your system of two first order differential equations, you can compute $x'(t)$ and $y'(t)$ for the results when $a = 0.02$, $b = 0.024$ and when $a = 0.02$, $b = 0.015$. From these values you can find the jumper's speed at landing. How do these speeds compare with the takeoff speed?

Exercise 3: A Better Model.

A more accurate model for ski jumping is one in which the drag and lift forces have magnitude proportional to the square of the jumper's speed. With that assumption, the second order differential equations might be a nonlinear system of the form

$$x''(t) = S(-ax'(t) - by'(t))$$
$$y''(t) = S(bx'(t) - ay'(t)) - 32.2$$
$$S = \sqrt{x'^2 + y'^2}$$

The values for a and b will be different for this system (the linear system (2) essentially replaced the variable speed with an average speed of the jumper for the duration of the jump). Moreover, it is no longer possible to simply integrate each of these equations with respect to t to obtain two first order differential equations. This system could be transformed, however, into a system of four first order differential equations. If your software will handle a system of four first order differential equations, experiment with this system to see what appropriate values would be chosen for a and b to obtain reasonable flight trajectories for the ski jumper.

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Heating and Cooling of Buildings

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(Editor's Note: Dunbar's heat experiment is based on the material of Section 3.3 of ***Fundamentals of Differential Equations****, R.K. Nagle and E.B. Saff (Addison-Wesley, 1992, 3rd Ed.). Though Dunbar's students use* ***Mathematica****, the experiment adapts easily to other software.)*

Our goal is to formulate and analyze a mathematical model that describes the 24-hour temperature variation inside a building. The interior variation will result from the outside temperature variation and the heat generated by the people and machines inside the building. We ignore the heating and cooling of the interior with furnaces or air-conditioning, so the situation modeled would best apply in spring or fall.

We know from Newton's Law of Cooling that if:

a) $T(t) =$ temperature inside the building as a function of the time t,

b) $M(t) =$ outside air temperature as a function of time,

c) $H(t) =$ internal temperature gain due to people, machinery, lights, etc., then

$$T' = K(M(t) - T(t)) + H(t) .$$

The factor K depends on the physical properties of the building, such as the number of doors and windows, and the type of insulation, but does not depend on T, M or t.