you can find the jumper's speed at landing. How do these speeds compare with the takeoff speed?

Exercise 3: A Better Model.

A more accurate model for ski jumping is one in which the drag and lift forces have magnitude proportional to the square of the jumper's speed. With that assumption, the second order differential equations might be a nonlinear system of the form

$$x''(t) = S(-ax'(t) - by'(t))$$
$$y''(t) = S(bx'(t) - ay'(t)) - 32.2$$
$$S = \sqrt{x'^2 + y'^2}$$

The values for a and b will be different for this system (the linear system (2) essentially replaced the variable speed with an average speed of the jumper for the duration of the jump). Moreover, it is no longer possible to simply integrate each of these equations with respect to t to obtain two first order differential equations. This system could be transformed, however, into a system of four first order differential equations. If your software will handle a system of four first order differential equations, experiment with this system to see what appropriate values would be chosen for a and b to obtain reasonable flight trajectories for the ski jumper.

References:

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Heating and Cooling of Buildings

Steven R. Dunbar

University of Nebraska
Lincoln, NE 12345
srd@mathcml.unl.edu

(Editor's Note: Dunbar's heat experiment is based on the material of Section 3.3 of ***Fundamentals of Differential Equations****, R.K. Nagle and E.B. Saff (Addison-Wesley, 1992, 3rd Ed.). Though Dunbar's students use* ***Mathematica****, the experiment adapts easily to other software.)*

Our goal is to formulate and analyze a mathematical model that describes the 24-hour temperature variation inside a building. The interior variation will result from the outside temperature variation and the heat generated by the people and machines inside the building. We ignore the heating and cooling of the interior with furnaces or air-conditioning, so the situation modeled would best apply in spring or fall.

We know from Newton's Law of Cooling that if:

a) $T(t) =$ temperature inside the building as a function of the time t,

b) $M(t) =$ outside air temperature as a function of time,

c) $H(t) =$ internal temperature gain due to people, machinery, lights, etc., then

$$T' = K(M(t) - T(t)) + H(t) .$$

The factor K depends on the physical properties of the building, such as the number of doors and windows, and the type of insulation, but does not depend on T, M or t.

We will assume that the air temperature varies as a sine function over a 24-hour period with it's minimum at $t = 0$ (midnight) and it's maximum at $t = 12$ (noon); so that

$$M(t) = M_0 - B\cos\omega t$$

where B is a positive constant and

$$\omega = \frac{2\pi}{24} = \frac{\pi}{12} .$$

We will also assume that $H(t) \equiv H_0$, so that the internal rate of temperature gain is constant. This assumes that the heat gain due to lights and machinery running all the time is a constant, a reasonable assumption.

1) By hand, show that the equation can be written in the standard linear equation form :

$$T' + KT = Q(t)$$

where K is a constant and

$$Q(t) = K(B_0 - B\cos\omega t) .$$

Note that B_0 is another constant, derived as a combination of M_0, H_0 and K.

2) Assume that at midnight, September 21 (the beginning of fall), the air-conditioning is turned off and the building temperature is 72°, so the assumptions above apply. Assume that for the next three days, the temperature varies from 50° at midnight to 90° at noon in the sinusoidal fashion assumed above. Let the constant $K = 1/6$ for a fairly well closed and insulated building. Take $H_0 = 1$ for a modest heat gain. It is not an oversight that I do not give you M_0 and B. You must determine these from the information given. Determine the equation for the building under these circumstances. Write the corresponding initial value problem with all parameters converted to the appropriate numeric values.

3) By using the **Mathematica** expressions for solving a first-order linear equation, attempt to solve the differential equation using this "easy automatic formula". Speculate on why **Mathematica** returns the answer that it does. Can you use this answer?

4) Use **Mathematica** to solve the differential equation.

a) Write the solution in mathematical form.

b) Graph the building temperature and the air temperature over a period of three days (72 hours) on one graph.

c) By inspecting your graph, find for each of the three days the maximum and minimum temperature in the building and the time at which they occur.

5) Explain the time difference between the external and internal temperature variations.

6) Based on your solution from part (4a) above, could you have predicted the results of part (5) without reference to the graphs? Examining your solution from (4a), what parameters should you vary in an effort to make the building more comfortable for the occupants. Explain your recommendations and show your results graphically. ❑

The Lorenz Attractor
Thirty years later

Courtney Coleman

Harvey Mudd College
Claremont, CA 91711
coleman@sif.claremont.edu

Determinism or Chaos: Who would have believed in 1963 that a newly published article in a technical journal would lead to a