$$C' = \{ R - C [S + F + A \sin (2\omega (t - 75)) - B \sin (\omega (t - 135))] \}$$

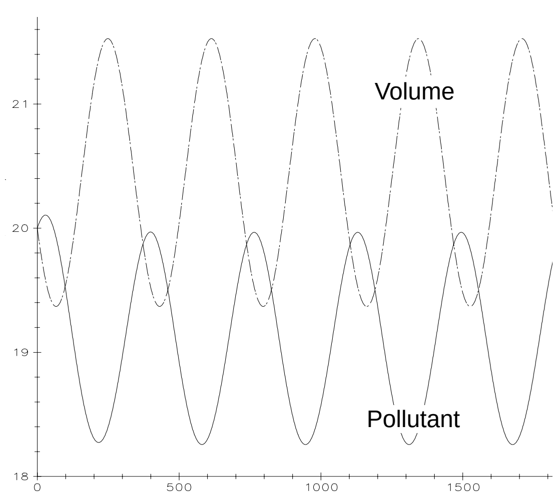
$$F = r_1 + r_2 + r_3 + r_4 + P - Q$$

Let $C_1 = 3 \times 10^{-4}$ Dm/day, $C_2 = 5 \times 10^{-5}$ $R_1 = 2.5 \times 10^{-4}$, and $R_2 = 4 \times 10^{-5}$, and we have that $F = 0$!

Sample Problems: The following problems should be done using an ODE solver. Attach graphics.

- 1. Set up and solve this system with the given parameter values. Let t go from 0 to 1825 days ($\cong 5$ years), with $\Delta t = 2$ days. Assume an initial concentration of 20 parts/cu.Dm for the pollutant in the lake.
- 2. Describe how the concentration of the pollutant varies over time. Does it reach some kind of steady state? Explain.

(Editor's Note: I used a local software package to solve this problem. Here you can see the regular rise and fall of pollution levels in the lake and the simultaneous change in lake volume.)



- 3. How would the equations change if the rainwater contains a pollutant concentration of p_r parts/cu.Dm Let p_r= 5, and redo #1 and #2.
- 4. Which is more damaging, a doubling of the pollutant level in stream 2 or in stream 3? (Try both cases, p_r = 0 and p_r = 5). Provide graphs and an explanation.

5. Make up a totally new hypothesis of your own, and test it. Provide graphs to back up your conclusions, and state carefully any changes you make in the parameters. ❑

Discovering Differential Equations

Margie Hale

Stetson University
DeLand, FL 32720
mhale@stetson.bitnet

While teaching at Ithaca College I had the opportunity to develop and teach a differential equations course in a laboratory equipped with ten Sparc workstations. Each class contained about ten students majoring in various liberal arts disciplines. The approach I took was to replace some calculation with modeling and qualitative analysis, and my lecturing with student discovery.

The principal software available was **MATHEMATICA**, chosen by the department to enhance many of the upper level mathematics courses. I wrote some programs specifically for a differential equations course, and I supplemented with BASIC programs for certain applications. No knowledge of computers was assumed at the start of the course.

Syllabus: Allowing students to experiment and to discover is a lengthy process.

The syllabus was pared down to include only three major topics: first order equations, second order (mostly linear) equations, and (not necessarily linear) systems. But we were able to cover these topics in some depth, and the students were encouraged to articulate the meaning of symbols and processes. For example, in the unit on first order equations, only two methods of exact solution were studied: separation of variables and integrating factors. Topics covered that I have omitted in previous courses were the Euler and Runge-Kutta approximation methods, direction fields, phase-time plots for autonomous equations, and stability. We also investigated several applications: population dynamics, other compartment problems, motion with resistance, and center of mass. In each case, the students, with some direction, developed the model themselves.

Class Structure: We met three days per week in the laboratory. The typical class would begin with my giving a short introduction to the topic of the day. The students then completed a “worksheet” designed to suggest explorations and lead them to one or more useful facts or techniques. They each had a computer, but could choose to work in pairs. Two or three activities during the semester were designed for groups of four or five. I wandered around watching them work and discussed their work with them individually. At the end of the class I gave a short summary of what they (hopefully) had done, and made connections to previous work.

Several class periods were devoted to physics experiments such as spring-mass and pendulum. The students took measurements and compared them later to the ideal models.

Homework reinforced and extended material developed in class. We used a text, and occasionally the student was assigned to read the author’s development or summary of what we had just investigated in class.

At the beginning I designed some activities especially to acquaint the students with the computer and software. After that the activities were driven by the goal of learning about differential equations in a modern technological environment. In other words, using the computer was never a goal, only a means. I was careful to include discussions of why a differential equation was an appropriate attack on a certain kind of problem, what we hoped to learn from the equation, and what were the alternative ways to analyze the equation.

A Logistic Modeling Problem: Following is an example of a modeling problem from the unit on first order equations. When the students began this project they had already developed the exponential and logistic population models, used the exponential model, and had experience with Euler’s method. Incidentally, I obtained the data from a biology professor studying bacteria. I discovered from him that this is a sure-fire way to get good data, as he says the biology experiment ends when the growth is no longer exponential.

Class Exercise 1. The given data consist of seven points covering six hours of bacteria growth. **MATHEMATICA** plots the data points versus time. Students are asked to guess the appropriate model. Students solve the equation by hand and approximate the parameters. Students enter their solution and the software graphs it and the data points on the same graph. Students adjust parameters until they are satisfied with the fit. Students are asked to choose a model for longer-term growth, and they discuss the need for different assumptions.

Homework 1. Students do problems

from the text on using the logistic equation and estimating parameters from data. (No computer is necessary.)

Class Exercise 2. Students are asked to fit a logistic model to the original bacteria data. They must choose two data points to help them compute parameters. Different choices give different parameters. Again, **MATHEMATICA** shows both graphs together. They experiment until they like the fit. Students predict the population at time 12, then compare with the "real" answer which I provide. The class discusses problems with real data and the process of parameter estimation.

Homework 2. Students use the Euler approximation method, available through **MATHEMATICA**, to solve the logistic equation. The software also graphs this numerical solution, and students compare it to the plot of the data points. Students discuss advantages and disadvantages of exact and approximate solutions to this problem.

Testing: I gave only two tests: a midterm and a final. The midterm was in two parts. In part 1 the students could use no references or tools; they were asked to solve some equations exactly, set up some models without solving, and to choose the "best" investigation technique for each in a list of differential equations. For part 2 of the midterm the students were in the laboratory, and could use their text, notes, and the computer. The questions were experimental in nature, and there were several ways students could approach them. For example, they had to draw several phase-time diagrams corresponding to a logistic population model with varying degrees of hunting, and discuss the effects of hunting. Some students used the computer to solve and graph the equations, either exactly or approximately. Others used only algebra to discover the maximum hunting level which allows the population to survive.

The final exam was in one part only, given in the laboratory. Some problems were done best with the computer, some not.

Now that I am Older and Wiser: Anyone who has brought students and computers together is certainly older and probably wiser than when they began. Here are some of the things I learned as a result of preparing and teaching this course.

- What I prepare as student activity never works just right the first time. I did extensive testing and revising of materials before the course began. Essentially, I worked leisurely over a summer to prepare the hand-outs for the course. On a second teaching, revisions were minor and easy to do.
- Students are slow. Plan for false starts and long delays while students sort out what you are asking them. Practice patience. My response was the relaxed pace of the syllabus described above.
- Computers are ornery. They occasionally give wrong answers and sometimes refuse to work at all. Know your software and its bugs. Keep a couple of non-computer activities handy for emergencies.
- Computers are excellent at holding students' attention, much better than I am. I found it best to talk in short, well-defined and isolated spurts, and to be quiet while students were working. (This is good advice for students who you hope are thinking, as well as those who are pressing keys.) One-on-one conversations initiated by the student worked well.

Student Assistants: Two pieces of advice which I took from the CASE newsletter were valuable. Both are related to keeping intellectual control in the classroom. First, I hired a computer-savvy student assistant to be in the laboratory during every class period. The assistant can respond to the student who pushes the wrong button while you're talking, leaving you free to direct the rest of the class. The assistant can call for help when the system crashes while you continue to focus the class on the subject matter. Second, each class period choose a student to sit at the machine connected to the projector instead of your sitting there. This way you can have demonstrations and still not interrupt the flow of your message.

Structuring The Laboratory: Here is a final word on the laboratory environment. The laboratory at Ithaca College is roomy, with each computer sitting on a desk which allows room for books and notebooks too. The instructor can get to every station for a personal discussion with the one or two students there, and to see their work. The monitors are lowered a few inches so that the students can easily see the professor and each other. These are all good points which contribute to a positive learning environment. The major flaw of that laboratory (as of 1992) was its lack of air conditioning. Computers like cold, and (being ornery) generate heat, and air conditioning is expensive.

Why MATHEMATICA? MATHEMATICA was chosen, after much deliberation, for its power and its adaptability to a variety of courses. It is superb at drawing single graphs, slower at drawing multiple graphs in color. As far as I know, it cannot draw graphs in "real time": you see only the finished product. This is a definite disadvantage in drawing phase-planes; one would like to see the dynamics in a dynamical system. The BASIC programs I wrote mostly addressed this problem. (A few were written so the students would have access to the code.) Probably one of the other software systems is better for use in a differential equations class only.

MATHEMATICA syntax is not friendly for the beginning computer student, and I had to provide a three-page reference sheet to help students with the commands they would need for the course.

Afterword: I am currently interested in collecting samples of specific questions that instructors ask their students in a computer environment. These should be questions which help a student make the connection between computer output and abstract idea. If you wish to share such questions, from any subject matter, please send them to me at the address above.

(Editor's Note: Hale's request is one that touches all of us teaching ODEs. Send **C•ODE•E** *a copy of your suggestions for possible publication.)* ❑

Differential Equations, Football, and Chase Problems

Steven R. Dunbar

University of Nebraska-Lincoln

Lincoln, NE 68588

srd@mathcml.unl.edu

I first learned of this problem in an article in the CASE Newsletter by Elgin Johnson and Jerold Mathews [1], describing the ``Project Based Calculus" at Iowa State University. It's also a classic problem which can be found in the text by Simmons [2].

College football is a favorite American