Review of PHASER

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There may be good reasons not to start up a computer lab for your ODEs course. Thanks to **PHASER**, however, the time and trouble of teaching students how to use the software isn't one of them.

PHASER, which bills itself as "An Animator/Simulator for Dynamical Systems for IBM Personal Computers", was developed by Huseyin Kocak to provide a computer laboratory component for the elementary ODE course at Brown University. The software is on 3 1/2" and 5 1/4" disks in Kocak's manual and reference book, **Differential and Difference Equations Through Computer Experiments** [Springer-Verlag, 1989 (2nd ed.)].

Phaser is for teaching ODEs: It's a classroom tool, not a research package. It can solve systems of up to nine 1st order ODEs, but only using Euler's Method, the improved Euler's Method, or 4th order Runge-Kutta (nonadaptive). The package comes with a library of many built-in equations.

Once an equation is entered the fun begins. The best thing about **PHASER** is that it is EASY! Students can be taught how to do computations and display results in no time, and the menu-driven displays allow for experimentation with other commands. Virtually all the commands come with default parameters, so that students can do something with a single keystroke, then refine their results as they learn more. My experience with **PHASER** has been that it requires virtually no office-hour support (with the possible exception of learning how to enter your own equations, **PHASER**'s weakest point).

There are many display options, including phase plane, component graphs, direction fields, Poincaré maps, rotations, etc. Equations can be entered with up to 12 parameters, which can later be varied to study bifurcations and other effects of changing parameters.Solutions of ODEs may also be computed in backward time.

Screens (and the corresponding settings) can be saved and later restored for use in lectures. The main menus contain a number of other useful features.

... and Difference Equations: **PHASER** handles difference equations of up to nine dimensions, and can compute and display stair-step diagrams. I used **PHASER** for my dissertation on discrete dynamical systems and obtained many interesting graphs to illustrate the theorems.

... and Basic Numerical Algorithms: Personally, I find the choice of numerical algorithms in **PHASER** to be just right. In an introductory ODEs course, one often begins the discussion of numerical methods with the original numerical algorithm -- Euler's method. All of the traps that more sophisticated algorithms can fall into can be seen in Euler, or at least in the (two-point averaged) improved Euler's method. Yet many fancy software packages provide only very smart adaptive solvers which solve most problems well. That's great for research -- but not for education!

It's important to me that an ODEs tool for classroom use should allow the demonstration of

Euler's method and the improved Euler's method, because that's what I'm trying to teach in an introductory course -- the ideas behind the basic methods, and the potential pitfalls of numerical methods in general. It would be great to have the option of using a sophisticated solver in **PHASER**, but having the basic methods is, in my opinion, much better. Besides, you can't fit everything on a single IBM PC diskette.

The Bestiary: No review of **PHASER** would be complete without mentioning the "manual". In addition to 15 well-designed tutorial lessons and a reference guide to the menus which contains just the right amount of detail, the book contains a short review of differential and difference equations and a short section on numerical methods in general. These are meant to supplement, not replace, a standard text on ODEs. Additionally, all of the stored equations that come with **PHASER** appear in a bestiary of equations and illustrations in the back of the book. This section includes not just the equation and the standard parameter settings, but also a short discussion of the equation's significance, and references to the literature. The value of this manual /reference book cannot be overemphasized.

The Low Down: If you're looking for computational power and a package that will grow with the student through more advanced follow-on courses, this probably is not the package for you. This is not a package for finding accurate answers to difficult problems, but if you want students with relatively little computer experience up and running on a package fast, this may be your best choice. You will need very little lecture time to explain the package.

If you want good software that can help your students learn about the geometry of solutions of ODEs and difference equations, software to introduce them to numerical methods, then **PHASER** may just be your ticket. ❑

Demonstrating the Stability of the Lagrange Points

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The goal of this lab is to make accessible to students of mechanics or differential equations the problem of determining the stability of motion of three bodies under mutual gravitational attraction. I have focused upon the restricted three-body problem, which is usually discussed in the later chapters of any text on analytical mechanics, such as Symon (**Mechanics**, Addison-Wesley, 1960).

Sun - Planet - Asteroid: The problem consists of describing the motion of a body of relatively small mass, M3, as it moves through the gravitational field of two bodies of significantly larger mass, M1 and M2, moving in circular orbits in a plane about their common center of mass (CM). An example of this would be the relationships between Jupiter, the sun, and the group of Trojan asteroids.

The problem is simplified by adopting a non-inertial frame of reference which rotates at the