A final PLANETS run (Figure 7) uses the setup in Figure 5, but this time the initial velocity values v_1 and v_2 are changed slightly so that the exact circular orbits of M1 and M2 become ellipses of low eccentricity. M3 oscillates about L4 in a leisurely fashion, with a substantial amplitude.

Figure 6

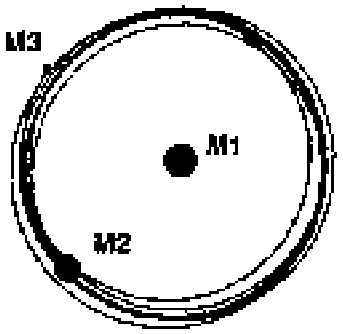
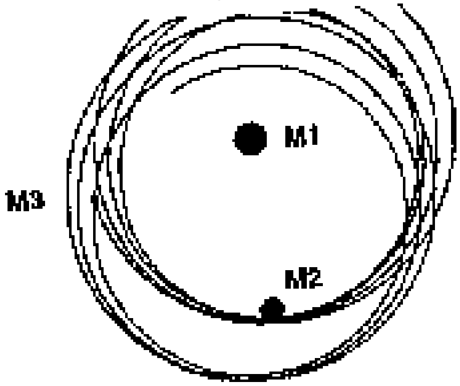


Figure 7



The demonstration described here is easy to carry out in the classroom, or could be used as a basis of student experiments with the restricted three body problem. The background may be easily adapted for a course in either ODEs or in classical mechanics. This lab is part of a project designed during a workshop on teaching ordinary differential equations at Cornell University.

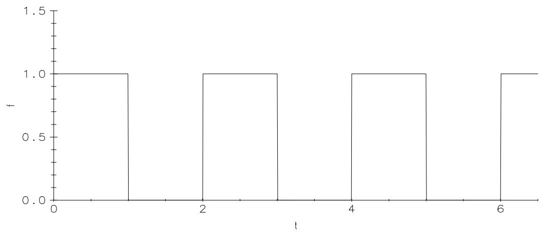
First Order Analysis of Communication Channels

Alan Felzer

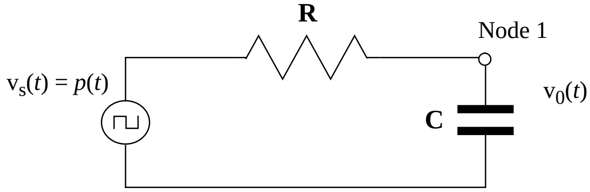
Cal State Polytechnic University

Pomona, CA 91733

Introduction: The availability of computers certainly makes it easier for students to explore and investigate differential equations - especially nonlinear ones like the logistic equation. This is great, but the computer is also useful in helping us to reinforce what's going on with linear differential equations, even first order ones with constant coefficients.



Let us consider the bread and butter problem in electrical engineering of determining the maximum rate at which a train of data pulses (a square wave)can be sent down a communications channel modeled by a first order equivalent circuit of the form



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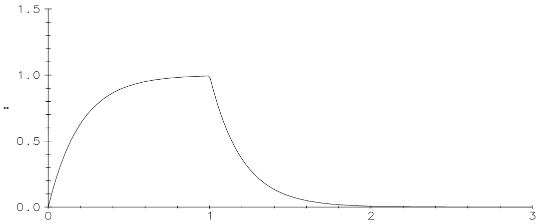
[1] **PLANETS** is included in the **MacMath** software distribution. For information, contact Beverly West at beverly@math.cornell.edu.ddd ❑

and still be “recognizable” at the output (especially in the presence of noise). The maximum rate (referred to as the data rate) is limited by how fast the capacitor can charge up when a data pulse turns on and then discharge when the data pulse turns off as indicated below.



The objective of this investigation is to see, in general terms, how the values of R and C affect the rate at which our simple circuit can respond to a pulse.

Circuit Equations: To model our circuit, we make use of Kirchhoff’s Current Law, which tells us that the current entering any given node must equal the current leaving the node. Doing this at node 1 in the circuit, we obtain that

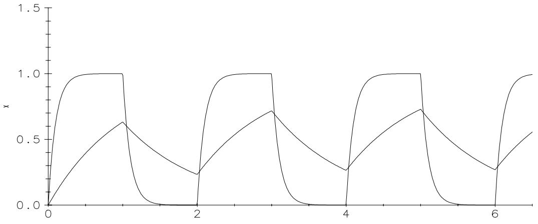
$$\frac{v_s(t) - v_0(t)}{R} = Cv_0'(t)$$

and so

$$v'_0 = -\frac{1}{RC}v_0(t) + \frac{1}{RC}v_s(t)$$

where $v_0(t)$ is the voltage across the capacitor and $v_s(t)$ is the source voltage. Then a solver is used to obtain plots of the circuit’s pulse train response as we vary the circuit parameters.

In particular, by varying R (keeping C constant) we obtain graphs such as this, from which we see that the larger the resistance becomes the more the pulses are distorted, and so the lower the channel’s data rate is going to be.



Conversely, the lower the circuit’s resistance, the more the output pulse train resembles the source pulse train in both amplitude and shape**.**

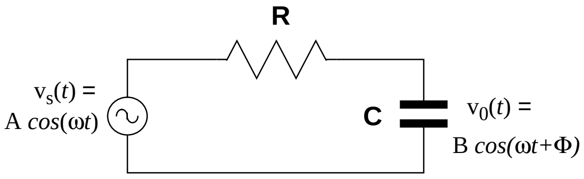
Pulse Train Response: Although formulas for the pulse train response can be developed, they are too complicated for an introductory course in ODEs. Students have a much easier time using solvers on computers to produce these pictures.

For a sinusoidal source (instead of the pulse train) simple formulas can be derived without too much trouble. From solutions to these or from computer solvers it is easy to show the effect of changing the resistance on the shape and the amplitude of the output voltage.

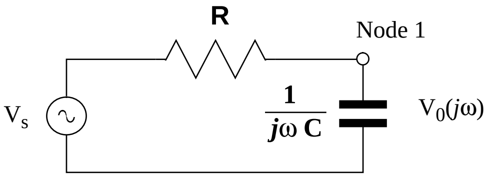
Of particular interest to electrical engineers is how the data rate is related to the channel’s frequency response, by which we mean the change in the magnitude B of the sinusoidal steady state response

$$v_0(t) = B\cos(\omega t + \Phi)$$

which varies as a function of the input frequency ω and the circuit parameters R and C. Electrical engineers like to relate a circuit’s properties to its frequency response because that is fairly easy to calculate and measure for linear circuits.



For example: To calculate the sinusoidal state response of our circuit we simply apply Kirchhoff’s Current Law as we did above but now to the phasor circuit (i.e., to the Fourier transform of the circuit and of the corresponding differential equation).



We obtain the algebraic equation

$$\frac{V_s - V_0(j\omega)}{R} = j\omega C V_0(j\omega)$$

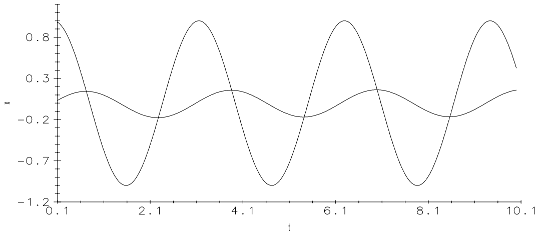
from which we obtain that

$$V_0(j\omega) = \frac{1}{1 + j\omega RC} V_s$$

Therefore, back in the time-domain

$$v_0(t) = \frac{A}{\sqrt{1 + (\omega RC)^2}} \cos(\omega t + \Phi)$$

Now, plotting the corresponding sinusoids at a given frequency ω for different values of R we obtain curves like these.



From these curves we see that the smaller the resistance, the larger the amplitude of the steady state response becomes. The correlation follows from the fact that the smaller the value of R, the larger the amplitude of the term

$$\frac{v_s(t)}{RC}$$

becomes in the ODE for v_0(t). As expected from the analysis of the response to the pulse train, the faster the circuit can respond to changes in the input.

In the language of electrical engineering, we say that as the bandwidth of a communication channel increases, the data rate increases (the bandwidth increases as resistance drops in the circuit). This is an important principle illustrated with a first order system, and could easily be discussed in a class on ODEs, especially with the aid of computers.

Editors Note: good ODE solvers should have square and triangular pulses and pulse trains as pre-defined functions. Felzer's article shows why these functions are useful. ❑

Buying a Computer Lab

David Lerner

University of Kansas

Lawrence, KS 66045

ILI, ODEs, Mathematica Notebooks: Last summer the Mathematics Department at the University of Kansas was awarded an ILI grant to establish a computer laboratory for our introductory ODE course. Our intention is to base the lab on a collection of Mathematica notebooks which we are writing. The existence of a suitable version of Mathematica is the only obvious constraint on the hardware. This article attempts to lay out the features we deemed most important and the reasons for our choice.

Our situation may not generalize to other institutions. Perhaps the most important