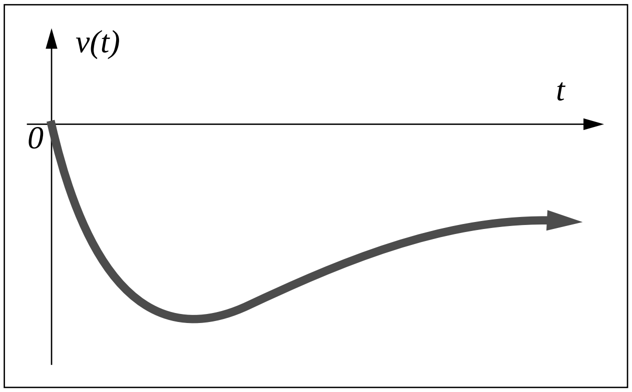
behavior in the solution. Try to explain why $f(v)$ produces the observed solution and what the resulting motion of the falling mathematician would look like.

Think Carefully: Do you think it is possible to choose an $f(v)$ which results in a graph of v versus t like that shown here?



In other words, is it possible for the falling mathematician to "exceed" terminal velocity?

Functions of Time Too! Finally, suppose our mathematician is falling through the air with wild updrafts and downdrafts. In this case, air resistance becomes a function of both v and t. Pick your own function $f(v,t)$ for air resistance, and see what strange behaviors you can generate. For example, can you find a function $f(v,t)$ for which the mathematician falls for a while and then starts to rise again? This might happen if the mathematician gets caught in a strong updraft. Be sure to keep a record of the functions you try and the results you obtain.

Instructors' Notes: This laboratory exercise is intended for students early in a first ODE course. The exercise will require a software package that supports graphical output. The students may have trouble finding the appropriate bounds for the systems, so guidance is suggested. The kinds of student responses to the questions posed in this lab are very much dependent on whether the students have been introduced to the ideas of equilibrium and the existence and uniqueness theorem. In particular, the "Think Carefully" problem requires an understanding of these matters. ❑

Book Review:

V.I. Arnol'd 's *Ordinary Differential Equations*

Stephen Kennedy

(Editor's Note: Kennedy reviews the third edition of one of the classic texts in the field, written by V.I. Arnol'd, and published by Springer-Verlag, 1992. The translation was done by R. Cooke, and the book sells for $49.00)

Near the turn of the century, Poincaré, trying to understand the n-body problem, began the qualitative study of differential equations, i.e., studying the stability, topology, and asymptotics of solutions rather than searching for analytical expressions for them. The Russian school of dynamical systems, beginning with Poincaré contemporary Lyapunov and continuing through Andronov, Pontryagin, Kolmogorov and Arnol'd, developed a beautiful, mature qualitative theory rooted in mechanics with a strong geometric flavor.

The book under review reflects this tradition. The geometric viewpoint is evident immediately. Chapter one, section one, is titled "Phase Space" and the concept is introduced by one of the loveliest problems in a book packed with great exercises:

Two non-intersecting roads lead from city A to city B. It is known that two cars traveling from A to B over different roads and joined by a cord of length less than 2L were able to travel from A to B without breaking the cord. Is it possible for two circular wagons of radius L whose centers move over these roads towards each other to pass without touching?

There are no mechanical exercises in this text – most ask for a proof or a significant insight. In fact, very few techniques for finding solutions are presented. Separation of variables is explained in three sentences and justified in the language of differential forms; variation of constants takes two pages.

After the introduction to phase space, direction fields, and integral curves, we get a baker's dozen of examples. These include: exponential and logistic growth (with and without harvesting), predator-prey systems, free fall, undamped pendula, and spherical pendula. The emphasis is always on qualitative properties and every example is illustrated. Most of the remainder of the chapter explores the notion of differential equation as flow on phase space. Changes of variable are naturally, and elegantly, described in this language. Chapter two contains the bones of the theory -- existence and uniqueness theorems, continuous and differential dependence on initial conditions and parameters, and an introduction to Lie algebras of vector fields. There is also a section on conservative vector fields. The heart of the book is the chapter on linear systems and their topological classification by spectra. We learn about asymptotic stability, Lyapunov functions, and in an extensive section on non-autonomous systems, Sturm-Liouville theory. The book concludes with two short chapters containing some previously deferred proofs and an introduction to differential equations on manifolds. Readers familiar with the landmark first edition will find many more examples, a handful of new topics, and an elegant new translation here.

It is difficult to imagine teaching a sophomore-level ODE course using this text, as Arnol'd does at Moscow University. The book requires mathematical sophistication and a familiarity with mechanics. There is no mention of series solutions, Laplace transforms, or numerical methods. On the other hand, it is a pure pleasure to read. The language is crisp and precise, without pedantry, there are no messy formulae, and there are 272 figures. The book is packed with beautiful ideas, including many that you never knew had a connection to differential equations: continued fraction expansions and estimating the square root of 2, the first digit of 2^n is more likely to be 7 than 8, the number of different ways to paint a cube. There is much here to enrich your ODE course, or even to fill a second semester! ❑