

The Fitzhugh-Nagumo Equations: A Model for Neural Activity

Background. The Fitzhugh-Nagumo equations are a model system for electrical activity in a neuron, an **excitable system**. A neuron can be stimulated with an input, such as an electric current. After the stimulus, the neuron is “excited.” The state of this **excitation** is described by the variable x , which represents the voltage (excitation) in the neuron as a function of time. When a neuron is excited, physiological processes in the cell will cause the neuron to **recover** from the excitation. The variable y in the model equations represents this recovery. The **stimulus** to the neuron is represented by z in the equations. A difference of step functions is used in the definition of $z(t)$ to represent the on-off switching of the stimulus. When the variable **off** = 0, the stimulus turns on at a prescribed time, and does not turn off. When **off** = 1, the stimulus turns off at a later time. The magnitude of the stimulus is measured by v . If $v = 0$, the neuron does not have an external stimulus. The Fitzhugh-Nagumo equations are:

$$x' = c \left(y + x - \frac{x^3}{3} \right) - z, \quad y' = \frac{a - by - x}{c}, \quad z = v[\text{step}(t, 30) - (\text{off})\text{step}(t, 60)].$$

Modeling Questions. Let’s study the response of the model neuron for various stimuli z . If the response of the neuron (x) tends to a steady state after a large displacement, we say that the neuron has **fired**, a so-called single **action-potential**. If the response of the neuron is a periodic oscillation of the voltage x , then the neuron experiences **repetitive firing**. Both kinds of behavior can be observed in real neurons.

When setting the parameters, be sure to use these restrictions in order to get meaningful behavior:

$$1 - \frac{2b}{3} < a < 1, \quad 0 < b < 1, \quad b < c^2.$$

A good choice is: $a = 0.75$, $b = 0.5$, $c = 1.0$.

- Study the model with no stimulus ($v = 0$) and describe how the un-stimulated neuron behaves.
- Explore the model with increasing values of v and describe your results. First study the model with **off** = 0 (i.e. stimulus stays on) and then study it with **off** = 1 (turn off the stimulus). Keep $0 \leq v \leq 1$. Under what conditions can you cause the neuron to repeatedly fire? If the stimulus to an excited neuron is removed, does the neuron return to its original state? When the neuron is repeatedly firing, does the period of the firing depend on the magnitude of the (constant) stimulus?

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/*FITZHUGH-NAGUMO MODEL ODES FOR
NEURAL ACTIVITY */
x' = c*(y + x - x^3/3) - z
y' = (a - b*y - x)/c
z = v*(step(t,30) - off*step(t, 60))

// x is the neural excitation (voltage)
// y is the recovery variable
// z is the stimulus to the neuron
// v is a constant; size of stimulus
//Set the parameters:
c = 1; b = .5; a = .75; v = .4
off = 0 /*set off = 1 to turn off
stimulus function. More ... */
    
```

Enter

Initial Conditions

1	0
x	3
y	0

Integration

Interval

Points

Clear

Equation set successfully solved

