Age of the Lascaux Cave Paintings

Background

In any living organism the ratio of the amount of ^{14}C to the total amount of carbon in the cells is the same as that in the air. After the organism is dead, ingestion of CO_2 ceases, and only the radioactive decay continues, at a rate proportional to the amount of ^{14}C in the sample. The half-life r of ^{14}C is known to be about 5568 years. Suppose that $x(t)$ is the amount of ^{14}C per gram of carbon at time t in the charcoal sample; $x(t)$ is dimensionless because it is the ratio of masses. Suppose that $t = 0$ is now, and that $T < 0$ is the time that the wood was burned. Then $x(t) = x(T) = x_T$ for $t \leq T$.

Suppose that x_0 is the amount of ^{14}C per gram of carbon in the sample at $t = 0$. Verify that on the interval $T \leq t \leq 0$, $x(t)$ is the unique solution of the backward IVP

$$\mathbf{x' = -kx, \quad x(0) = x_0, \qquad T \leq t \leq 0}$$

The *half-life*, τ, of ^{14}C is the amount of time required for half of the ^{14}C nuclei to decay (no matter what you start with). It can be shown that

$$\tau = (\ln 2)/k$$

Some Questions

In 1950 a Geiger counter was used to measure the decay rate of ^{14}C in charcoal fragments found in a cave near Lascaux, France, where there where there are prehistoric wall paintings of various animals. The counter recorded about 1.68 disintegrations per minute per gram of carbon, while for living tissue such as the wood in a tree the number if disintegrations was 13.5 per minute per gram of carbon. When the wood was burned to make the charcoal (and so determine the age of the cave paintings). [HINT: The reading of a Geiger counter at time t is proportional to $x'(t)$, the rate of decay of radioactive nuclei in a sample.]

(a) An analytical solution is generated by solving the IVP above to find

$$\mathbf{T = -(1/k)\ln(x_T/x_0) = -(\tau/\ln 2)\ln(x'(T)/x'(0))}$$

and then using the data in the problem statement to find T.

(b) A numerical approach to answering this question appears on the next page.

Radiocarbon Dating

- Problem

  Sample: 1.69 disinteg/gram carbon/min

  Living Tissue: 13.5 disinteg/gram carbon/min

  Half-life, τ, of $^{14}C = 5568$ years

- $x(t) =$ grams ^{14}C/grams carbon at time t

  $t = 0$: Present

  $t = T$: Time of Death ($T < 0$)

- $x_0 =$ grams ^{14}C/gram carbon at time 0 $(x_0 = x(0))$

  $x_T =$ grams ^{14}C/gram carbon at time T $(x_T = x(T))$

- Put $y = x/x_0$, then

$$\begin{aligned} y' &= \begin{cases} 0, & t < T \\ -ky, & t \geq T \end{cases} \\ y(0) &= 1 \end{aligned}$$

  or

$$\begin{cases} y' &= \text{if } y > 13.5/1.68 \text{ then } 0, \text{ else } -(\ln 2)y/5568 \\ y(0) &= 1 \end{cases}$$

- $T \sim 16,700$years

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/*RADIOCARBON DATING OF CAVE PAINTINGS
Charcoal found in the caves of Lascaux,
France, is used to date the paintings
found on the walls.  The ODE model for
the (dimensionless) amount y of radio-
active carbon-14 in the charcoal is:*/
     y' = -k*y
     k = (ln(2))/5568; u = 13.5/1.68
/*where y=Q(t)/Q(0) and Q(t) is the
the amount of C-14 per gram of char-
coal.  The ratio of the decay rate of
C-14 in living tissue to that in the
cave sample is 13.5/1.68, and 5568
years is the half-life of C-14.    */
```

Enter | Solve | Sweep | Extend

Initial Conditions

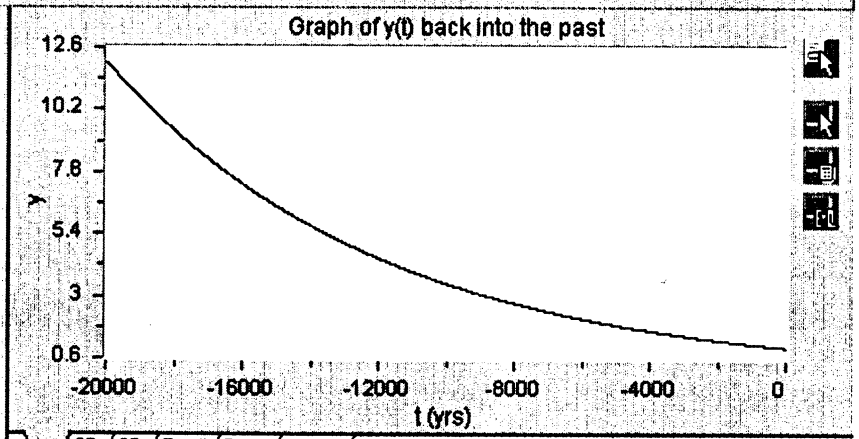
t	0
y	1

Integration

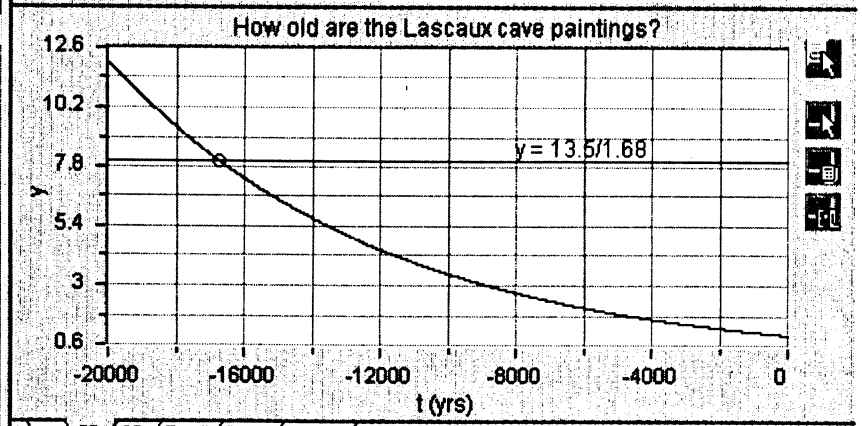
Interval	-5000
# Points	1000

Clear

IC / Sweep / Solver / Equilibrium



t-y / 2D / 3D / Equil / Data / Model



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Equation set successfully solved. 16:1

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